

Algorithmic Model Theory — Assignment 10

Due: Tuesday, 1 July, 12:00

Exercise 1

Give automatic presentations of the following structures:

- (a) $(\mathbb{N}, +)$, and
- (b) $(\mathbb{Q}, <)$ (recall Exercise 9.1).

Give an ω -automatic presentation of

- (c) (\mathbb{R}^+, \cdot) .


Exercise 2

Show that, for each $p \in \mathbb{N}$, the following structures are mutually interpretable, and give an automatic presentation of \mathcal{V}_p .

- $\text{Tree}_p = (\{0, \dots, p-1\}^*, s_0, \dots, s_{p-1}, <, \text{el})$, where $<$ is the prefix relation, and el is the equal-length predicate, i.e. $\text{el}(x, y)$ holds if x and y have the same distance from the root.
- $\mathcal{N}_p = (\mathbb{N}, +, |_p)$ where $x |_p y$ holds if there exists a k such that $x = p^k$ and $x | y$.
- $\mathcal{V}_p = (\mathbb{N}, +, V_p)$ where $V_p : \mathbb{N} \rightarrow \mathbb{N}$ is a function mapping x to the largest power of p dividing it.

Exercise 3

Give automatic presentations of the following orderings:

- (a) $(\mathbb{N}, <)$,
- (b) $(\mathbb{N} + \mathbb{N}, \triangleleft)$: 
 i.e., the structure consisting of two disjoint copies of \mathbb{N} such that $x \triangleleft y$ if both elements are in the same copy and $x < y$ (where $<$ is the natural ordering on \mathbb{N}), or if x is an element in the first copy and y is an element in the second copy, and
- (c) $(\mathbb{N} \times \mathbb{N}, <_{\text{lex}})$ where $<_{\text{lex}}$ is the lexicographical ordering derived from $<$.