Knowledge and Cooperation in Infinite Games

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Epistemology

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- Current form: Kripke structures

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- For our concerns justified!

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- Consider logics which allow to express temporal statements about the knowledge of the agents

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- The *knowledge* of agent i at some point (π, n) is given by $\sim_i$
- Equivalent points are indistinguishable for agent i, i.e., if $(\pi, n) \sim_i (\rho, m)$ then at point (π, n) and (ρ, m), agent i has exactly the same information, so he cannot distinguish one situation from the other

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$$\varphi ::= P \mid \varphi \wedge \varphi \mid \neg\varphi \mid X\varphi \mid \varphi U \varphi \mid K_i\varphi \mid$$

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$\mathcal{E} \models \varphi$ iff $\mathcal{E}, (\pi, 0) \models \varphi$ for all $\pi \in R$

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- *Run:* infinite sequence $\pi = v_0 a_1 v_1 \ldots \in V(AV)^\omega$ such that $(v_i, a_{i+1}, v_{i+1}) \in \Delta$ for each $i < \omega$

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Here we focus on synchronous perfect recall:

$$(\pi, n) \;\sim_i^*\; (\rho, m) \;\; :\Longleftrightarrow \;\; m = n \text{ and } a_j \sim_i^A b_j,\; v_j \sim_i^V w_j$$

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We also consider an instance of asynchronous perfect recall:

$$(\pi, n) \overleftarrow{\sim}_i^* (\rho, m) \;:\Longleftrightarrow\; (\overleftarrow{\pi}, n) \sim_i^* (\overleftarrow{\rho}, m)$$

where $\overleftarrow{\pi}$ is obtained from π by contracting each maximal sequence $v_r a_{r+1} v_{r+1} \ldots a_s v_s$ with $v_j \sim_i^V v_{j+1}$ to v_r

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Using this formalism, we want to check finite systems for epistemic properties:

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- R is the set of all runs of $\mathcal{E}$ from v
- $\zeta(\pi, n) = \{P \in \mathrm{Prop} \mid v_n \in P\}$
- $\sim_i$ is either $\sim_i^*$ or $\overleftarrow{\sim}_i^*$

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Polynomial Space
(Translation of LTL-formulas into Büchi automata)

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(2)
- Use k-trees to interpret formulas of knowledge-depth k
- For fixed k, infinite sequences of k-trees can be recognized by automata
- Also involves a factorization of formulas into temporal and knowledge components
- $\rightsquigarrow$ space polynomial in $|\varphi| \cdot \text{exp}(\text{depth}(\varphi), O(|\mathcal{E}|))$

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  Until and Common Knowledge allow an arbitrary reach through two orthogonal dimensions of the semantic structures
- For synchronous systems and formulas with common knowledge but *without until* PSPACE-complete (van der Meyden, Shilov, '99)
  Without until, the temporal operators can only look $|\varphi|$ steps into the system

From Verification to Control

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- We are not only interested in model checking closed systems but in *synthesizing reactive* systems
- Reactive Systems interact with an environment
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- Question: Can we ensure a faultless interaction of the components of the system, independently of the behavior of the environment
- Natural Model: Games on Graphs

Games with Partial Observation

Multi-player games on graphs:

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- A multi-player game is basically a multi-agent system, where each position is controlled either by one of the cooperating players or by the environment
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- Question: Does this coalition have a winning strategy for the game?
- The winning condition for the coalition is defined by the specification φ
- We often consider systems with partial observation
- It is then very desirable to be able to refer to the knowlegde of the components in the specification: ETL
- The environment may also have partial information about the system

Games with Partial Observation

A *game with* n *players* has the form

$$\mathcal{G} = (V, (V_i)_{i \in \underline{n}}, \mathrm{Prop}, \Delta, (\sim_i^V)_{i \in \underline{n}}, (\sim_i^A)_{i \in \underline{n}})$$

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where

- V is the finite set of positions
- V_i is the set of positions of player i
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- $\Delta \subseteq V \times A \times V$ is the move relation
- $\sim_i^V$ and $\sim_i^A$ are equivalence relations on V and A respectively

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1. $v \sim_i w \Rightarrow v, w \in V_i$ or $v, w \notin V_i$
2. $v \sim_i w \Rightarrow \mathrm{act}(v) = \mathrm{act}(w)$
3. $a, b \in A_i$ and $a \neq b \Rightarrow a \not\sim_i^A b$

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- *Winning strategy for the coalition:* Joint strategy σ such that each play π which is consistent with σ is won by the coalition

Synthesis from ETL-specifications

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An ETL-formula φ defines a winning condition

$$L(\varphi) = \{\pi \in (VA)^\omega \mid (\pi, 0) \models \varphi\}$$

Synthesis from ETL-specifications: Two Players

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- However: if $(\pi, n) \sim_0 (\rho, m)$ for $\sim_0 \in \{\sim_0^*, \overleftarrow{\sim}_0^*\}$, then (π, n) is consistent with σ_0 if, and only if, (ρ, m) is consistent with σ_0
- Due to the fact that player 0 can distinguish any two of his own actions

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- If player 0 models a controller of an environment which is not actually antagonistic but merely unpredictable, then given any strategy σ_0 for player 0, the joint system is constrained by σ_0, so the knowledge of both players should be relative to σ_0
- If player 0 models a network server which might interact with a user, then the protocol of the server may be off-limits to the user, so player 1 does not know σ_0
- One requirement for the protocol might be that the user is never able to learn the value of some internal variables of the server
- This can be expressed using K_1

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- So $\varphi =$ parity $\wedge\, G(\neg K_1 K_0^{\mathrm{col}} \wedge \neg K_1 \neg K_0^{\mathrm{col}})$ additionally requires that player 1 never knows whether player 0 knows the recent color

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Synthesis from ETL-specifications: Two Players

- For synchronous finite two-player games with ETL winning conditions which use only the knowledge operator K_0 2EXPTIME-complete (van der Meyden, Vardi '98)
- Uses tree automata: A tree automaton can process the tree-representation of a strategy, viewed as a function $\sigma_0 : \mathrm{Obs}_i^* \to A$
- Check that all plays are won by player 0: Universal part
- Evaluate K_0: Knowledge sets

Synthesis from ETL-specifications: Two Players

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Knowledge set of player i at point (π, n):

$$\mathcal{K}_i(\pi, n) = \{\text{last}(\rho, m) \,|\, (\rho, m) \sim_i (\pi, n)\} \subseteq V$$

(Set of positions that player i considers possible at point (π, n))

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Can be computed iteratively:

$$\mathcal{K}_i(\pi, n+1) = \mathrm{Post}_{[a_{n+1}]}(\mathcal{K}_i(\pi, n)) \cap [v_{n+1}]$$

$\rightsquigarrow$ **Powerset Construction** (Reif '84)

Transforms a two-player game with partial information into a two-player game with full information

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Theorem

Any ETL definable winning condition is ω-regular. ('10)

More precisely: For any game $\mathcal{G}$ and any ETL-formula φ with knowledge-operators K_0 and K_1 which have either $\sim_i^*$ or $\overleftarrow{\sim}_i^*$ semantics, we can effectively construct an S1S-formula $\psi(x)$ such that for any play π and any $n \in \mathbb{N}$ we have

$$(\pi, n) \models \varphi \iff \pi \models \psi(n)$$

S1S: Monadic Second Order Logic interpreted in word structures $(\mathbb{N}, (P_a)_{a \in \Sigma}, <)$

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- Also holds for the asynchronous case

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$$
\begin{aligned}
\neg\exists X_{va} \quad [\quad & \forall y(\bigvee_{va}(X_{va}y \wedge \bigwedge_{wb\neq va} \neg X_{wb}y)) \\
\wedge \quad & \forall y \forall z(Sy = z \rightarrow \bigvee_{(v,w)\in E_a,\, b\in A} X_{va}y \wedge X_{wb}z) \\
\wedge \quad & \forall (y \leq x)(\bigwedge_{va}(P_{va}y \rightarrow \bigvee_{wb\sim va} X_{wb}y)) \\
\wedge \quad & \neg\psi(P_{va}/X_{va}) \quad]
\end{aligned}
$$

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- A nondeterministic ω-automaton can guess such a (ρ, m) asynchronously
- In this particular asynchronous case the automaton can be constructed effectively

Synthesis from ETL-specifications: Two Players

Corollary

The asynchronous synthesis problem for ETL specifications with knowledge operators K_0 and K_1 is decidable.

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- For synchronous hierarchical n-player games with LTL winning conditions decidable (Pnueli, Rosner '90, van der Meyden, Wilke '05)
- For synchronous 3-player games with ETL winning conditions where only player 0 has partial observation undecidable (van der Meyden, Wilke '05)

Synthesis from ETL-specifications

On the other hand:

- Omega-Regularity of ETL winning conditions also holds for n agents (same proof)

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- such that the hierarchy of knowledge if preserved
- Parity Objectives are LTL-definable
- Hierarchical games with ETL winning conditions are decidable

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The actual question van der Meyden, Vardi and Wilke ask is

> Given a game $\mathcal{G}$ and an ETL specification φ, is there a joint strategy σ for the coalition such that $\mathcal{R}(\mathcal{G}, \sigma) \models \varphi$?

where $\mathcal{R}(\mathcal{G}, \sigma)$ is the unravelling of $\mathcal{G}$ with respect to σ!

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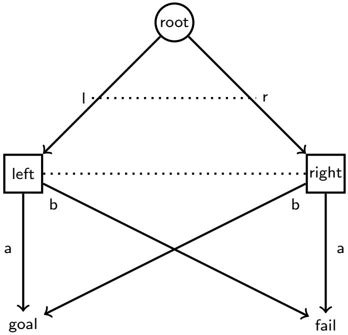
- There, the evaluation of knowledge operators is relative to plays which are consistent with σ
- So all players know the strategy of any other cooperating player
- In the two-player case this is equivalent but in the multiplayer case it makes a difference

Cooperation in Multi-Player Games

Knowing the strategies of your companions is different from *relying* on their strategies:



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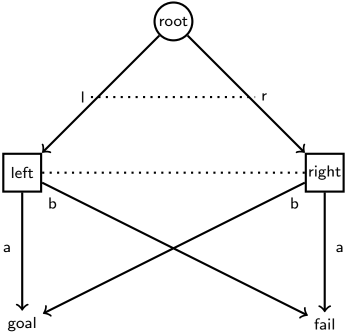
$\mathcal{K}_1 = \{\text{left}, \text{right}\} : \neg K_1 P_{\text{left}}$



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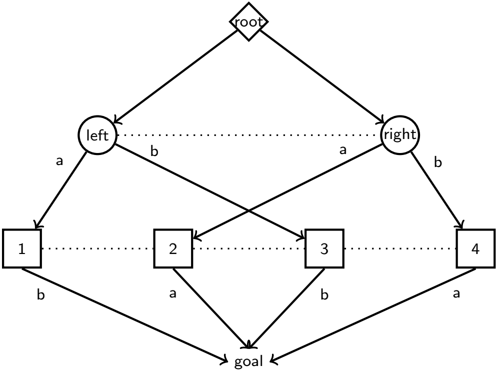

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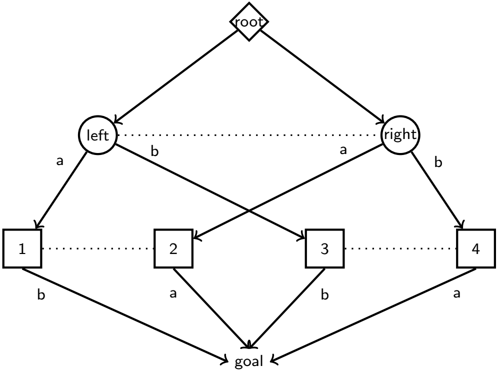

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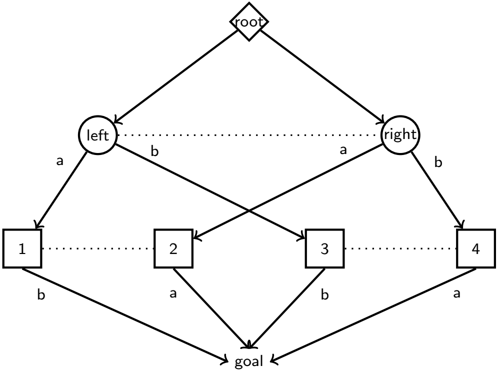

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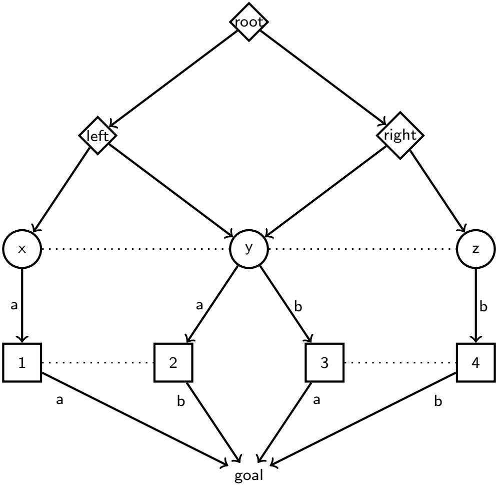

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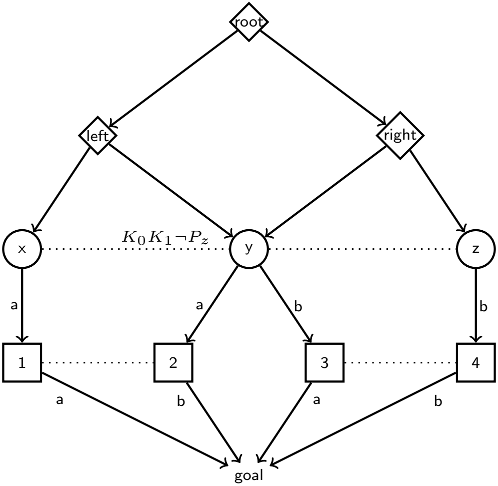
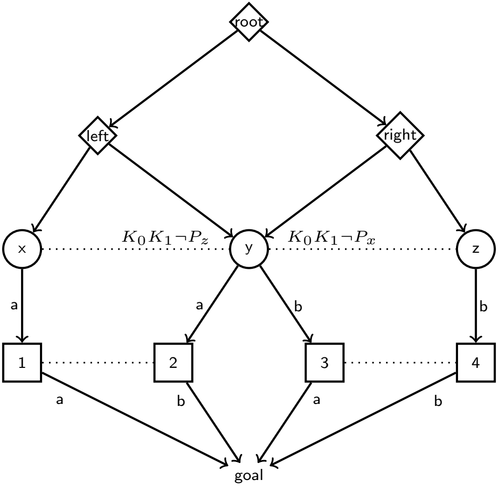


Cooperation in Multi-Player Games

Your knowledge-set does not provide you sufficient information, even if you *do know* the strategies of your companions:



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