Asynchronous Omega-Regular Games with Partial Information

MFCS, Brno, 23. Aug. - 27. Aug. 2010

Bernd Puchala

RWTH Aachen University

Algorithmic Synthesis of Reactive Systems

Basic Problem:

Algorithmic Synthesis of Reactive Systems

Basic Problem:

- Given a *plant* $\mathcal{P}$ and a *specification* φ
- Construct a *controller* $\mathcal{C}$ which ensures that all possible system behaviours satisfy φ

Algorithmic Synthesis of Reactive Systems

Basic Problem:

- Given a *plant* $\mathcal{P}$ and a *specification* φ
- Construct a *controller* $\mathcal{C}$ which ensures that all possible system behaviours satisfy φ
- System is modeled as *game*

controller (player 0) vs. environment (player 1)

Algorithmic Synthesis of Reactive Systems

Basic Problem:

- Given a *plant* $\mathcal{P}$ and a *specification* φ
- Construct a *controller* $\mathcal{C}$ which ensures that all possible system behaviours satisfy φ

- System is modeled as *game*

  controller (player 0) vs. environment (player 1)
- A *winning strategy* for player 0 yields a controller which ensures a correct system

Algorithmic Synthesis of Reactive Systems

Basic Problem:

- Given a *plant* $\mathcal{P}$ and a *specification* φ
- Construct a *controller* $\mathcal{C}$ which ensures that all possible system behaviours satisfy φ

- System is modeled as *game*

  controller (player 0) vs. environment (player 1)
- A *winning strategy* for player 0 yields a controller which ensures a correct system

Goals

Goals

Consider more general models for plants:
timed/hybrid/stochastic/distributed systems

Goals

Consider more general models for plants:
timed/hybrid/stochastic/distributed systems

Efficient algorithms:

Goals

Consider more general models for plants:
timed/hybrid/stochastic/distributed systems

Efficient algorithms: 16:20

Goals

Consider more general models for plants:
timed/hybrid/stochastic/distributed systems

Efficient algorithms: 16:20

- Handle more general specifications:
  - not necessarily observation based
  - knowledge operators

Goals

Consider more general models for plants:
timed/hybrid/stochastic/distributed systems

Efficient algorithms: 16:20

- Handle more general specifications:
  - not necessarily observation based
  - knowledge operators
- Explore new ways to model sources of uncertainties
  - asynchronous observability
  - bounded ressources/imperfect recall

Goals

Consider more general models for plants:
timed/hybrid/stochastic/distributed systems

Efficient algorithms: 16:20

- Handle more general specifications:
  - not necessarily observation based
  - knowledge operators
- Explore new ways to model sources of uncertainties
  - asynchronous observability
  - bounded ressources/imperfect recall
- Understand the information flow in systems
  - knowledge tracking
  - epistemic logics

The Model: Games

Nonterminating systems with finite state space

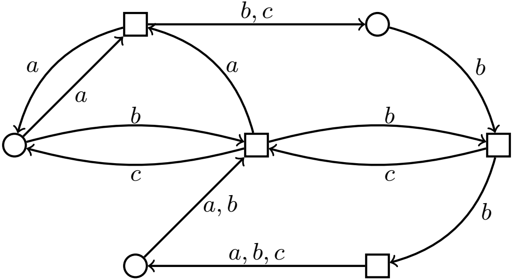
The Model: Games

Nonterminating systems with finite state space

$\rightsquigarrow$

Infinite games on finite graphs

$$G = (V, V_0, \delta : V \times A \to V, \varphi \subseteq (VA)^\omega)$$



The Model: Partial Information

Controller with restricted observational powers:

- imprecise sensors
- internal variables of some components
- ...

The Model: Partial Information

Controller with restricted observational powers:

- imprecise sensors
- internal variables of some components
- ...

- The controller has *partial information* about the states and the actions of the system
- Modeled by *equivalence relations*: Player 0 has the same information about equivalent positions/actions (*indistinguishability*)

The Model: Partial Information

Controller with restricted observational powers:

- imprecise sensors
- internal variables of some components
- ...

- The controller has *partial information* about the states and the actions of the system
- Modeled by *equivalence relations*: Player 0 has the same information about equivalent positions/actions (*indistinguishability*)

$$\mathcal{G} = (G, \sim^V, \sim^A)$$

The Model: Observability

What does player 0 observe as the system evolves over time?

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{aligned} \pi &= v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \rho &= w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{aligned}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \vdots \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \vdots \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho$$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$\pi \sim^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho$$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \vdots \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho$$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{aligned} \pi &= v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \rho &= w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{aligned}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \vdots \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$\pi \sim^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$$\pi \sim^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6 \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

What does player 0 observe as the system evolves over time?

Synchronous perfect recall: Player 0 observes any action

$\pi \sim^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 $(v \sim^V w)$ are completely hidden from player 0

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \to w$ of player 1 which have no visible effect for player 0 $(v \sim^V w)$ are completely hidden from player 0

$\pi \overleftarrow{\sim}^* \rho$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \rightarrow w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$$\pi \overleftarrow{\sim}^* \rho \qquad \begin{aligned} \pi &= v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9 \\ \rho &= w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{aligned}$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \to w$ of player 1 which have no visible effect for player 0 $(v \sim^V w)$ are completely hidden from player 0

$$\pi \overleftarrow{\sim}^* \rho$$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \rightarrow w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$$\pi \overleftarrow{\sim}^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9 \\ \vdots \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$\pi \overleftarrow{\simeq}^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$\pi \overleftarrow{\simeq}^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 $(v \sim^V w)$ are completely hidden from player 0

$\pi \overleftarrow{\simeq}^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \to w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$$\pi \overset{\leftarrow}{\simeq}{}^* \rho \qquad \begin{array}{l} \pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9 \\ \rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6 \end{array}$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \to w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$$\pi \overleftarrow{\eqsim}^* \rho$$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 ($v \sim^{V} w$) are completely hidden from player 0

$$\pi \overset{\leftarrow}{\eqsim}^{*} \rho$$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \to w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$\pi \overleftarrow{\simeq}^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 $(v \sim^V w)$ are completely hidden from player 0

$\pi \overleftarrow{\eqsim}^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Observability

Asynchronous Perfect Recall:
Moves $v \longrightarrow w$ of player 1 which have no visible effect for player 0 ($v \sim^V w$) are completely hidden from player 0

$\pi \overleftarrow{\eqsim}^* \rho$

$$\pi = v_0\, a_0\, v_1\, a_2\, v_2\, a_3\, v_3\, a_4\, v_4\, a_5\, v_5\, a_6\, v_6\, a_7\, v_7\, a_8\, v_8\, a_9\, v_9$$

$$\rho = w_0\, b_0\, w_1\, b_2\, w_2\, b_3\, w_3\, b_4\, w_4\, b_5\, w_5\, b_6\, w_6$$

The Model: Strategies

Partial information strategy for player 0 with respect to $\sim \in \{\sim^*, \overleftarrow{\sim}^*\}$:

The Model: Strategies

Partial information strategy for player 0 with respect to $\sim \in \{\sim^*, \overleftarrow{\approx}^*\}$:

$$f : (VA)^* V_0 \to A \text{ with}$$

$$\pi \sim \rho \qquad \Longrightarrow \qquad f(\pi) = f(\rho)$$

Solving Partial Information Games on Graphs

How to solve partial information games on graphs?

Solving Partial Information Games on Graphs

How to solve partial information games on graphs?

- We have to keep track of the knowledge of player 0 during the course of events

Solving Partial Information Games on Graphs

How to solve partial information games on graphs?

- We have to keep track of the knowledge of player 0 during the course of events
- Tree Automata
  implicit tracking: strategy as tree

Solving Partial Information Games on Graphs

How to solve partial information games on graphs?

- We have to keep track of the knowledge of player 0 during the course of events
- Tree Automata
  implicit tracking: strategy as tree
- Powerset Construction (Reif '84)
  explicit tracking: knowledge sets

Solving Partial Information Games on Graphs

How to solve partial information games on graphs?

- We have to keep track of the knowledge of player 0 during the course of events
- Tree Automata
  implicit tracking: strategy as tree
- Powerset Construction (Reif '84)
  explicit tracking: knowledge sets

Explicit knowledge tracking helps to understand the structure of information in games:
approach to represent all the information which is needed as succinctly as possible

Solving Partial Information Games on Graphs

How to solve partial information games on graphs?

- We have to keep track of the knowledge of player 0 during the course of events
- Tree Automata
  implicit tracking: strategy as tree
- Powerset Construction (Reif '84)
  explicit tracking: knowledge sets

Explicit knowledge tracking helps to understand the structure of information in games:
approach to represent all the information which is needed as succinctly as possible
Hierarchical Multiplayer Games can be solved with tree automata, but the approach of explicit knowledge tracking reveals an important difference between two and multiple players

Powerset Construction

Powerset Construction

- Generalization of Powerset Construction to arbitrary winning conditions

Powerset Construction

- Generalization of Powerset Construction to arbitrary winning conditions
- Preservation of omega-regular conditions

Powerset Construction

- Generalization of Powerset Construction to arbitrary winning conditions
- Preservation of omega-regular conditions
- Adaption to the Asynchronous case

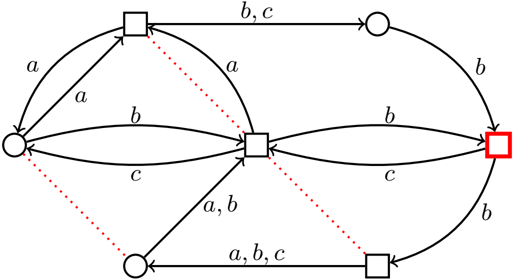
Powerset Construction

- Generalization of Powerset Construction to arbitrary winning conditions
- Preservation of omega-regular conditions
- Adaption to the Asynchronous case

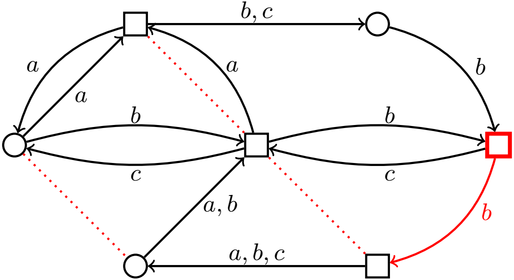
Theorem

Asynchronous ω-regular games with partial information are decidable and finite memory strategies suffice to win.

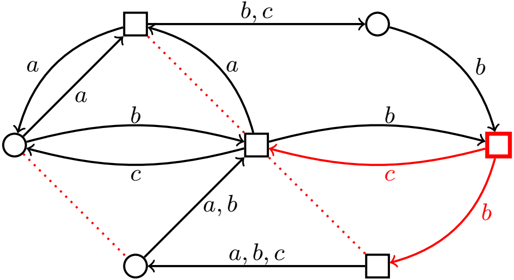
Knowledge Tracking



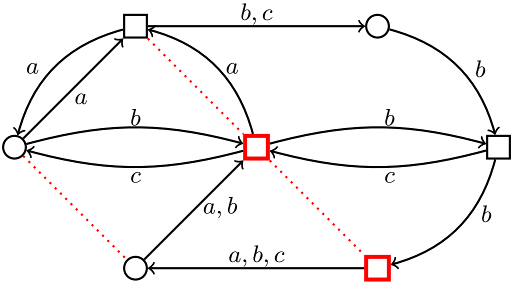
Knowledge Tracking



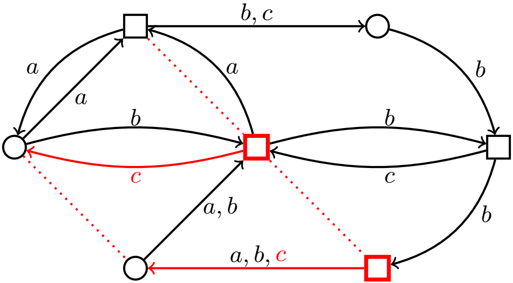
Knowledge Tracking



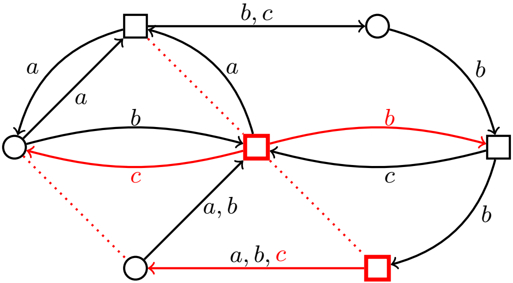
Knowledge Tracking



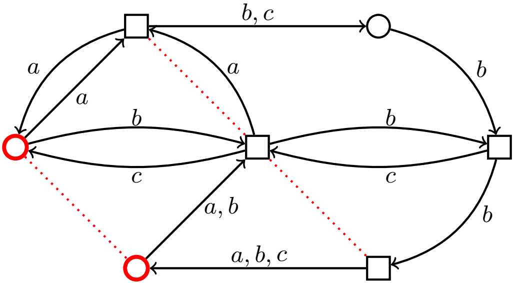
Knowledge Tracking



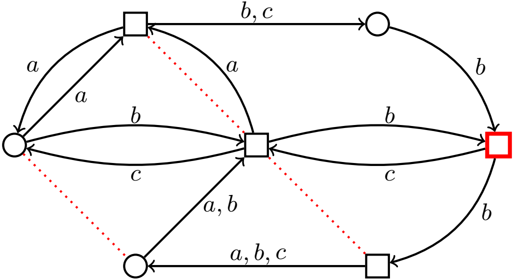
Knowledge Tracking



Knowledge Tracking



Knowledge Tracking



Powerset Construction

- From a given initial position, construct complete tracking of $\mathcal{G}$
- Yields a new game graph $\mathcal{G}^{\text{tr}}$ where the positions are subsets of positions of $\mathcal{G}$
- In $\mathcal{G}^{\text{tr}}$, player 0 has full information

Powerset Construction

- From a given initial position, construct complete tracking of $\mathcal{G}$
- Yields a new game graph $\mathcal{G}^{\text{tr}}$ where the positions are subsets of positions of $\mathcal{G}$
- In $\mathcal{G}^{\text{tr}}$, player 0 has full information
- Strategies can be translated from $\mathcal{G}$ to $\mathcal{G}^{\text{tr}}$ and back directly

Powerset Construction

- From a given initial position, construct complete tracking of $\mathcal{G}$
- Yields a new game graph $\mathcal{G}^{\text{tr}}$ where the positions are subsets of positions of $\mathcal{G}$
- In $\mathcal{G}^{\text{tr}}$, player 0 has full information
- Strategies can be translated from $\mathcal{G}$ to $\mathcal{G}^{\text{tr}}$ and back directly
- How to define the winning condition?

Parity Conditions with Observable Colors

- Parity condition is given by a coloring $\mathrm{col} : V \to \{1, \ldots, k\}$
- Player 0 wins, if the least color seen infinitely often is even
- If col is constant over equivalence classes of positions, then we can define a parity condition for $\mathcal{G}^{\mathsf{tr}}$ directly
- For $\overline{u} \subseteq [v]$: $\quad \mathrm{col}(\overline{u}) = \mathrm{col}(u)$ for some $u \in \overline{u}$

Arbitrary Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad ?$$

Arbitrary Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad ?$$

- $\overline{\pi}$ represents a set $\mathsf{Tr}^{-1}(\overline{\pi})$ of plays in $\mathcal{G}$
- $\pi = v_0 a'_1 v_1 a'_2 v_2 \ldots$ such that, for all $i < \omega$
  - $v_i \in \overline{v}_i$
  - $a'_{i+1} \sim^A a_{i+1}$

Arbitrary Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad ?$$

- $\overline{\pi}$ represents a set $\mathsf{Tr}^{-1}(\overline{\pi})$ of plays in $\mathcal{G}$
- $\pi = v_0 a'_1 v_1 a'_2 v_2 \ldots$ such that, for all $i < \omega$
  - $v_i \in \overline{v}_i$
  - $a'_{i+1} \sim^A a_{i+1}$
- Player 1 can enforce $\overline{\pi}$ against a strategy of player 0 for $\mathcal{G}^{\mathsf{tr}}$ iff he can enforce any $\pi \in \mathsf{Tr}^{-1}(\overline{\pi})$ against the corresponding strategy of player 0 for $\mathcal{G}$

Arbitrary Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad ?$$

- $\overline{\pi}$ represents a set $\mathsf{Tr}^{-1}(\overline{\pi})$ of plays in $\mathcal{G}$
- $\pi = v_0 a'_1 v_1 a'_2 v_2 \ldots$ such that, for all $i < \omega$
  - $v_i \in \overline{v}_i$
  - $a'_{i+1} \sim^A a_{i+1}$
- Player 1 can enforce $\overline{\pi}$ against a strategy of player 0 for $\mathcal{G}^{\mathsf{tr}}$ iff he can enforce any $\pi \in \mathsf{Tr}^{-1}(\overline{\pi})$ against the corresponding strategy of player 0 for $\mathcal{G}$

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\, a_0\, \overline{v}_1\, a_2\, \overline{v}_2\, a_3\, \overline{v}_3\, a_4\, \overline{v}_4\, a_5\, \overline{v}_5 \cdots \models \varphi^{\mathsf{tr}}$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\, a_0\, \overline{v}_1\, a_2\, \overline{v}_2\, a_3\, \overline{v}_3\, a_4\, \overline{v}_4\, a_5\, \overline{v}_5\, \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi})$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} \;= \overline{v}_0\, a_0\, \overline{v}_1\, a_2\, \overline{v}_2\, a_3\, \overline{v}_3\, a_4\, \overline{v}_4\, a_5\, \overline{v}_5\, \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi \;= v_0\, a_0'\, v_1\, a_2'\, v_2\, a_3'\, v_3\, a_4'\, v_4\, a_5'\, v_5\, \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} \; = \overline{v}_0 \, a_0 \, \overline{v}_1 \, a_2 \, \overline{v}_2 \, a_3 \, \overline{v}_3 \, a_4 \, \overline{v}_4 \, a_5 \, \overline{v}_5 \, \cdots \models \varphi^{\mathsf{tr}}$$

$$\uparrow$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi \; = v_0 \, a'_0 \, v_1 \, a'_2 \, v_2 \, a'_3 \, v_3 \, a'_4 \, v_4 \, a'_5 \, v_5 \, \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\, a_0\, \overline{v}_1\, a_2\, \overline{v}_2\, a_3\, \overline{v}_3\, a_4\, \overline{v}_4\, a_5\, \overline{v}_5 \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi = v_0\, a_0'\, v_1\, a_2'\, v_2\, a_3'\, v_3\, a_4'\, v_4\, a_5'\, v_5 \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} \ = \overline{v}_0 \ a_0 \ \overline{v}_1 \ a_2 \ \overline{v}_2 \ a_3 \ \overline{v}_3 \ a_4 \ \overline{v}_4 \ a_5 \ \overline{v}_5 \ \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi \ = v_0 \ a'_0 \ v_1 \ a'_2 \ v_2 \ a'_3 \ v_3 \ a'_4 \ v_4 \ a'_5 \ v_5 \ \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} \;= \overline{v}_0\; a_0\; \overline{v}_1\; a_2\; \overline{v}_2\; a_3\; \overline{v}_3\; a_4\; \overline{v}_4\; a_5\; \overline{v}_5\; \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi \;= v_0\; a'_0\; v_1\; a'_2\; v_2\; a'_3\; v_3\; a'_4\; v_4\; a'_5\; v_5\; \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\, a_0\, \overline{v}_1\, a_2\, \overline{v}_2\, a_3\, \overline{v}_3\, a_4\, \overline{v}_4\, a_5\, \overline{v}_5 \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi = v_0\, a'_0\, v_1\, a'_2\, v_2\, a'_3\, v_3\, a'_4\, v_4\, a'_5\, v_5 \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\ a_0\ \overline{v}_1\ a_2\ \overline{v}_2\ a_3\ \overline{v}_3\ a_4\ \overline{v}_4\ a_5\ \overline{v}_5\ \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi = v_0\ a'_0\ v_1\ a'_2\ v_2\ a'_3\ v_3\ a'_4\ v_4\ a'_5\ v_5\ \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\begin{array}{ll} & \overline{\pi} \;= \overline{v}_0\ a_0\ \overline{v}_1\ a_2\ \overline{v}_2\ a_3\ \overline{v}_3\ a_4\ \overline{v}_4\ a_5\ \overline{v}_5 \cdots \models \varphi^{\mathsf{tr}} \\ & \qquad\ \uparrow \quad \vdots \quad \uparrow \quad \vdots \quad \uparrow \quad \vdots \quad \uparrow \\ \forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad & \pi \;= v_0\ a'_0\ v_1\ a'_2\ v_2\ a'_3\ v_3\ a'_4\ v_4\ a'_5\ v_5 \cdots \end{array}$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\ a_0\ \overline{v}_1\ a_2\ \overline{v}_2\ a_3\ \overline{v}_3\ a_4\ \overline{v}_4\ a_5\ \overline{v}_5\ \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi = v_0\ a'_0\ v_1\ a'_2\ v_2\ a'_3\ v_3\ a'_4\ v_4\ a'_5\ v_5\ \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} \;= \overline{v}_0\; a_0\; \overline{v}_1\; a_2\; \overline{v}_2\; a_3\; \overline{v}_3\; a_4\; \overline{v}_4\; a_5\; \overline{v}_5\; \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi \;= v_0\; a'_0\; v_1\; a'_2\; v_2\; a'_3\; v_3\; a'_4\; v_4\; a'_5\; v_5\; \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} = \overline{v}_0\ a_0\ \overline{v}_1\ a_2\ \overline{v}_2\ a_3\ \overline{v}_3\ a_4\ \overline{v}_4\ a_5\ \overline{v}_5\ \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi = v_0\ a'_0\ v_1\ a'_2\ v_2\ a'_3\ v_3\ a'_4\ v_4\ a'_5\ v_5\ \cdots$$

Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\overline{\pi} \; = \overline{v}_0 \; a_0 \; \overline{v}_1 \; a_2 \; \overline{v}_2 \; a_3 \; \overline{v}_3 \; a_4 \; \overline{v}_4 \; a_5 \; \overline{v}_5 \; \cdots \models \varphi^{\mathsf{tr}}$$

$$\forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \qquad \pi \; = v_0 \; a_0' \; v_1 \; a_2' \; v_2 \; a_3' \; v_3 \; a_4' \; v_4 \; a_5' \; v_5 \; \cdots$$

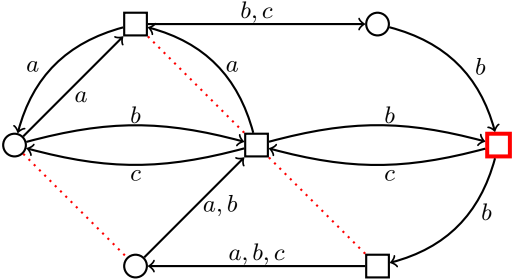
Omega-Regular Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

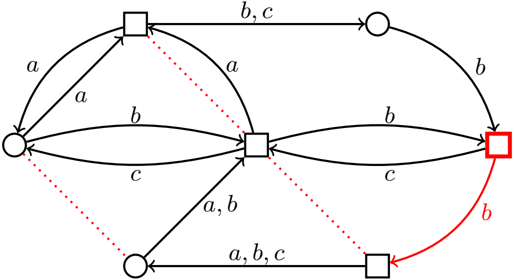
- If W_0 is omega-regular, then there is an S1S-formula φ with $L(\varphi) = W_0$
- New formula says: for any partition of ω into sets X_{va}, labelling ω in such a way, that the resulting ω-word π is in $\mathsf{Tr}^{-1}(\overline{\pi})$ we have $\pi \models \varphi$

$$\begin{aligned} & \overline{\pi} = \overline{v}_0\, a_0\, \overline{v}_1\, a_2\, \overline{v}_2\, a_3\, \overline{v}_3\, a_4\, \overline{v}_4\, a_5\, \overline{v}_5 \cdots \models \varphi^{\mathsf{tr}} \\ \forall \pi \in \mathsf{Tr}^{-1}(\overline{\pi}) \quad & \pi = v_0\, a'_0\, v_1\, a'_2\, v_2\, a'_3\, v_3\, a'_4\, v_4\, a'_5\, v_5 \cdots \models \varphi \end{aligned}$$

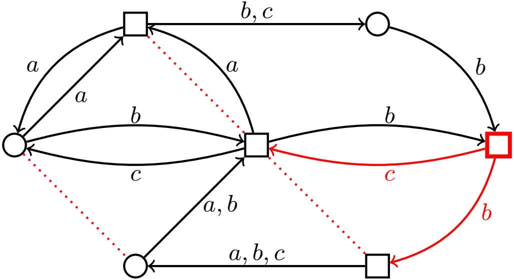
Knowledge Tracking



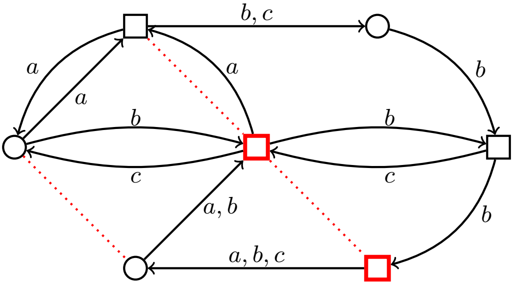
Knowledge Tracking



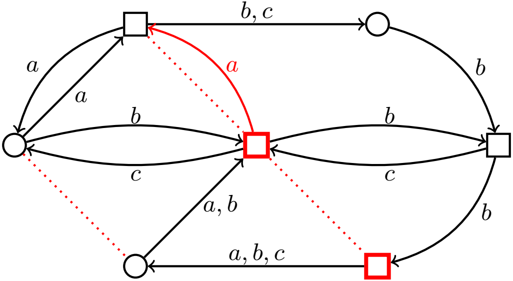
Knowledge Tracking



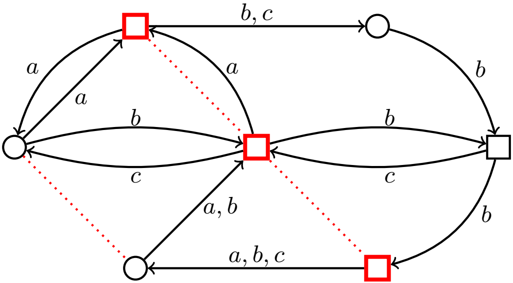
Knowledge Tracking



Knowledge Tracking



Knowledge Tracking



Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

Winning Conditions

$$\overline{\pi} = \overline{v}_0 a_1 \overline{v}_1 a_2 \overline{v}_2 \ldots \in W_0^{\mathsf{tr}} \quad :\Longleftrightarrow \quad \mathsf{Tr}^{-1}(\overline{\pi}) \subseteq W_0$$

$\pi = v_0 a'_1 v_1 a'_2 v_2 \ldots \in \mathsf{Tr}^{-1}(\overline{\pi}) \quad :\Longleftrightarrow$
there are numbers $0 = k_0 < k_1 < \ldots$ such that, for all $i < \omega$

$$v_{k_i}, \ldots, v_{k_{i+1}-1} \in \overline{v}_i$$
$$a'_{k_{i+1}} \sim^A a_{i+1}$$
$$k_{i+1} - k_i = 1, \text{ if } \overline{v}_i \subseteq V_0$$

Omega-Regularity

- Not clear how to express $\pi \in \mathsf{Tr}^{-1}(\overline{\pi})$ directly in S1S
- A priori unboundedly long sequences of private moves

Omega-Regularity

- Not clear how to express $\pi \in \mathsf{Tr}^{-1}(\overline{\pi})$ directly in S1S
- A priori unboundedly long sequences of private moves
- Use automata!

Omega-Regularity

- Not clear how to express $\pi \in \mathsf{Tr}^{-1}(\overline{\pi})$ directly in S1S
- A priori unboundedly long sequences of private moves
- Use automata!
- Consider nondeterministic Büchi automaton $\mathcal{A}$ with $L(\mathcal{A}) = W_0^c$
- Construct a nondeterministic Büchi automaton $\mathcal{A}^{\mathsf{tr}}$ which
  - reads $\overline{\pi}$
  - guesses a $\pi \in \mathsf{Tr}^{-1}(\overline{\pi})$
  - checks that $\pi \notin W_0$ by simulating $\mathcal{A}$ on π

Omega-Regularity

$((p, v, i), \tilde{v}a, (q, w, j)) \in \Delta^{\text{tr}} \quad :\Longleftrightarrow$

there is a finite history $v_1 a_2 v_2 \ldots a_n v_n$ in G such that:

1. $n = 1$, if $\tilde{v} \in \tilde{V}_0$
2. $v_1 = v$ and $v_l \in \tilde{v}$ for all $1 \leq l \leq n$
3. there is some $b \in act(v_n)$ with $b \sim^A a$ and $f_b(v_n) = w$
4. there are $q_1, \ldots, q_{n-1} \in Q$ such that
   - 4.1 $(p, v_1 a_2, q_1), \ldots, (q_{n-2}, v_{n-1} a_n, q_{n-1}), (q_{n-1}, v_n b, q) \in \Delta$
   - 4.2 $q_l \in F$ for some $0 < l < n$ if $j = 1$.

Omega-Regularity

$((p, v, i), \tilde{v}a, (q, w, j)) \in \Delta^{\mathrm{tr}} \quad :\Longleftrightarrow$

there is a finite history $v_1 a_2 v_2 \ldots a_n v_n$ in G such that:

1. $n = 1$, if $\tilde{v} \in \tilde{V}_0$
2. $v_1 = v$ and $v_l \in \tilde{v}$ for all $1 \leq l \leq n$
3. there is some $b \in act(v_n)$ with $b \sim^A a$ and $f_b(v_n) = w$
4. there are $q_1, \ldots, q_{n-1} \in Q$ such that
   - 4.1 $(p, v_1 a_2, q_1), \ldots, (q_{n-2}, v_{n-1} a_n, q_{n-1}), (q_{n-1}, v_n b, q) \in \Delta$
   - 4.2 $q_l \in F$ for some $0 < l < n$ if $j = 1$.

Show that $((p, v, i), \tilde{v}a, (q, w, j)) \in \Delta^{\mathrm{tr}}$ is decidable!

Results

Theorem

Asynchronous ω-regular games with partial information are decidable and finite memory strategies suffice to win.

Results

Theorem

Asynchronous ω-regular games with partial information are decidable and finite memory strategies suffice to win.

Theorem

Epistemic temporal formulas with synchronous and asynchronous knowledge operators for both players can be effectively translated into S1S-formulas.

Results

Theorem

Asynchronous ω-regular games with partial information are decidable and finite memory strategies suffice to win.

Theorem

Epistemic temporal formulas with synchronous and asynchronous knowledge operators for both players can be effectively translated into S1S-formulas.

Corollary

Asynchronous games with partial information and ETL winning conditions are decidable and finite memory strategies suffice to win.