

Directed Graphs of Entanglement Two

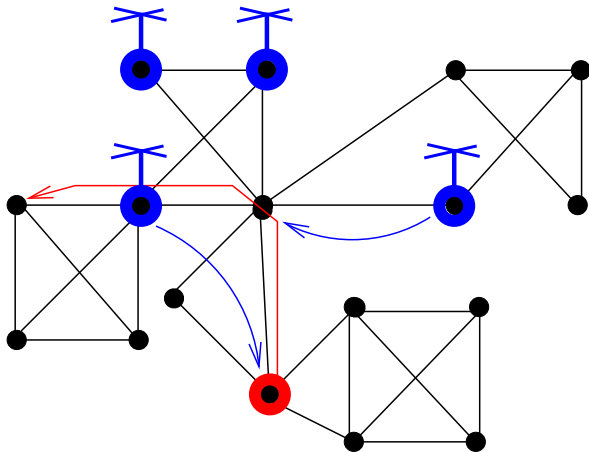
Roman Rabinovich

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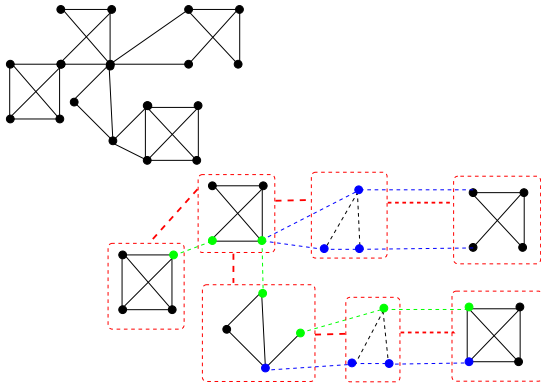
GAMES

Warsaw, 08.09.08

Treewidth



Tree decomposition



Complexity Measures for Directed Graphs

Measures can be expressed as **searching games**:

directed treewidth: (Johnson, Robertson, Seymour, Thomas)

- robber runs along **directed** paths within **SCCs**
- decomposition: directed tree

DAG-width: (Berwanger, Dawar, Hunter, Kreutzer; Obdržálek)

- robber runs along **directed** paths
- decomposition: DAG

Kelly-width: (Hunter, Kreutzer)

- robber runs along **directed** paths
- **invisible, inert** robber
- decomposition: DAG

Entanglement

Motivation: variable hierarchy of μ -calculus (Berwanger, Grädel, Lenzi)

Game Rules:

- in every move, the robber **must** go along an edge
- one cop can **only follow** her in a helicopter

No characterisation via decomposition known!

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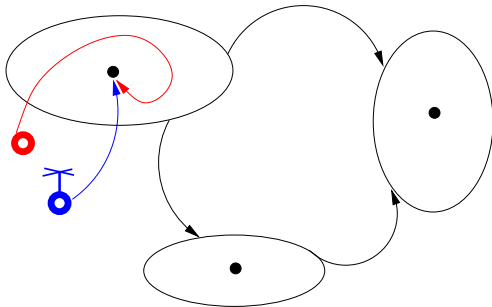
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Graphs of Entanglement 0 and 1

Entanglement is **zero** iff the graph is acyclic

Entanglement is ≤ 1 iff in each SCC \mathcal{C}

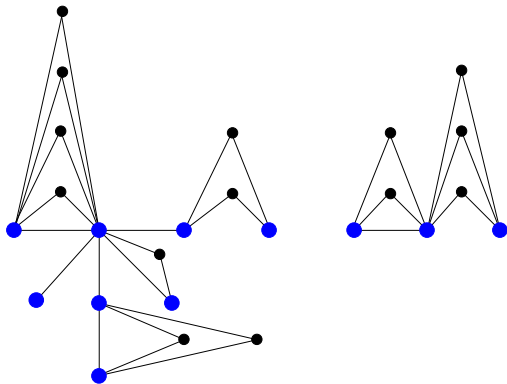
there exists vertex v with $\mathcal{C} \setminus v$ acyclic



Undirected Graphs with Entanglement at most Two

Theorem (Belkhir, Santocanale, 2007)

An undirected graph has entanglement at most two iff the graph has the form

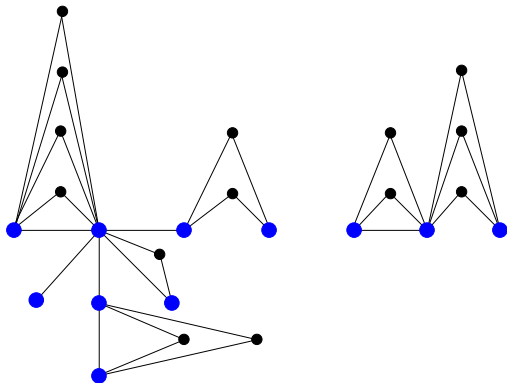


We generalise this result to directed graphs.

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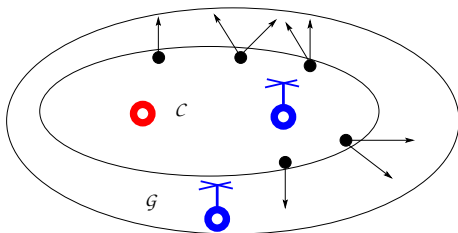
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Simple and Complex Subgraphs

\mathcal{C} : strongly connected subgraph of \mathcal{G}

\mathcal{C} is **simple** if

- (1) $ent(\mathcal{C}) \leq 1$, **or**
- (2) Cop player wins Entanglement Game with exit points



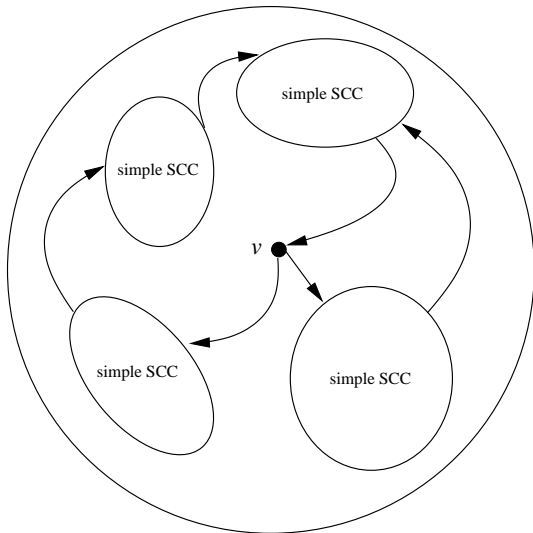
otherwise **complex**

Main Result

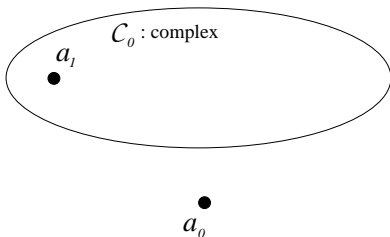
Theorem

Entanglement of a strongly connected directed graph \mathcal{G} is at most two iff there is a vertex v s.t. every SCC of $\mathcal{G} \setminus v$ is simple.

Simple Direction



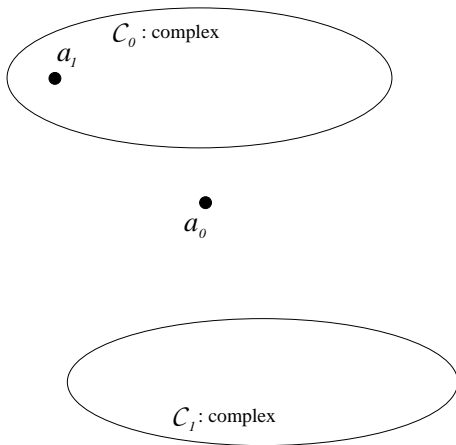
The Other Direction



Assumption:

- Entanglement ≤ 2 , but
- for all v there exists complex SCC of $\mathcal{G} \setminus v$.

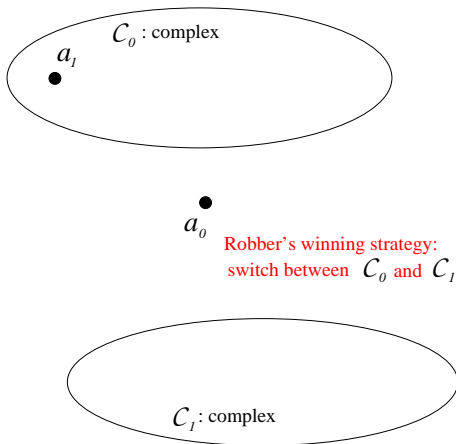
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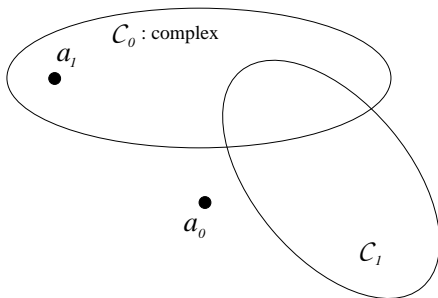
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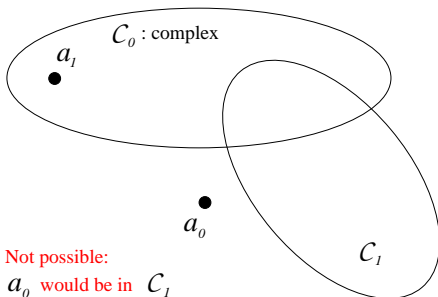
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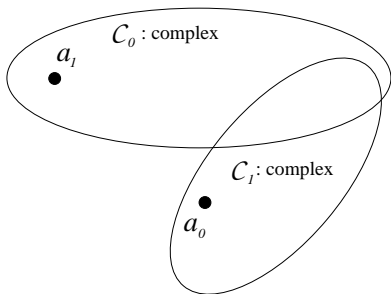
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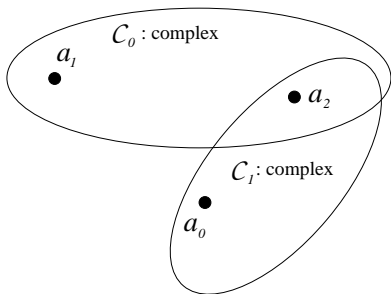
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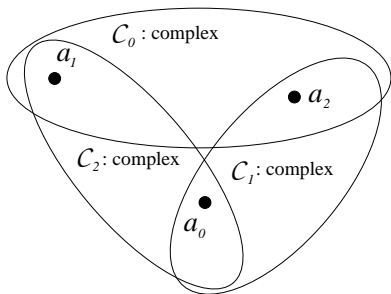
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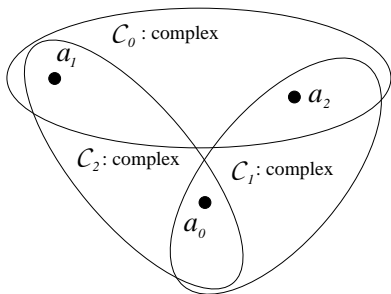
Triangle Case



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Triangle Case



Robber's winning strategy:
move between C_0 , C_1 and C_2

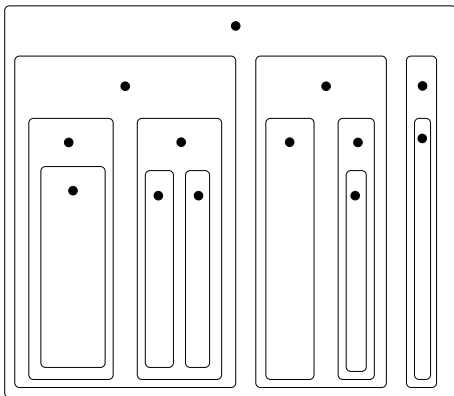
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Decomposition for Entanglement Two

Corollary

Every directed graph of entanglement at most two has a tree-like structure.



Comparison with Kelly-width and DAG-width

Theorem

If a graph has entanglement at most two then it has

- *DAG-width at most three.*
- *Kelly-width at most three.*

Conclusions, Outlook

- Goal: a characterisation of entanglement as a decomposition to construct efficient algorithms.
- Done: a decomposition up to entanglement 2.
- Future work: a generalisation of the modular approach
 - the notion of simple component
 - find a Robber's winning strategy in the triangle-case:

