

# Graph Searching Games with Multiple Robbers and Games with Imperfect Information

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# Complexity Measures

What is a simple graph?

- ▶ a small graph
- ▶ can be described by certain formalisms
  - ▶ in certain logics (e.g., FO, LTL)
  - ▶ by graph grammar
  - ▶ by an inductive construction (e.g., clique-decomposition)
  - ▶ algebraically (e.g., (bi-)rank-width)
  - ▶ ...
- ▶ with few intertwined cycles
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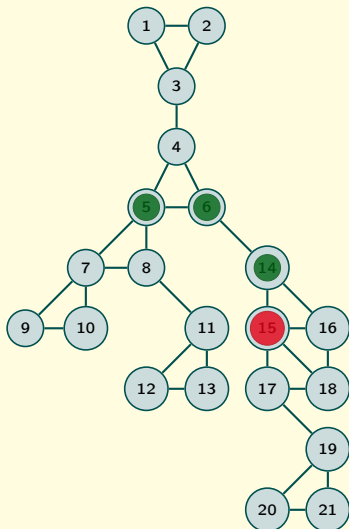
# Cyclicity Measures

- ▶ tree-width and path-width [Robertson, Seymour]
- ▶ directed tree-width [Johnson, Robertson, Seymour, Thomas]
- ▶ DAG-width [Berwanger, Dawar, Hunter, Kreutzer; Obdržálek]
- ▶ directed path-width [Reed, Seymour, Thomas]
- ▶ Kelly-width [Hunter, Kreutzer]
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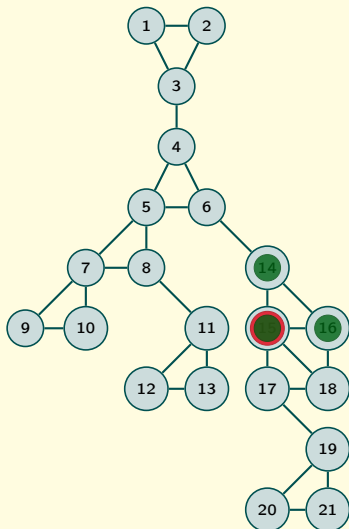
# Tree-width: Game Theoretical Definition



Game rules:

- ▶  $k$  Cops, one Robber
- ▶ Robber runs along cop free paths
- ▶ Cops fly
- ▶ Cops want to capture Robber

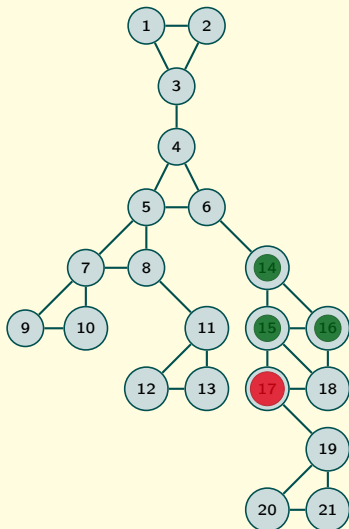
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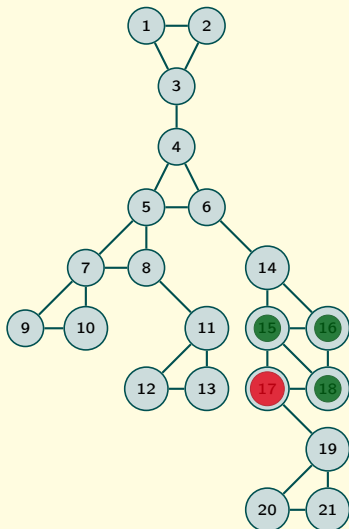
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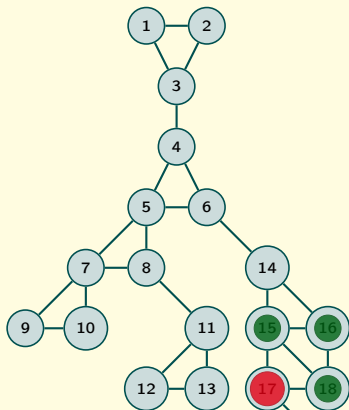
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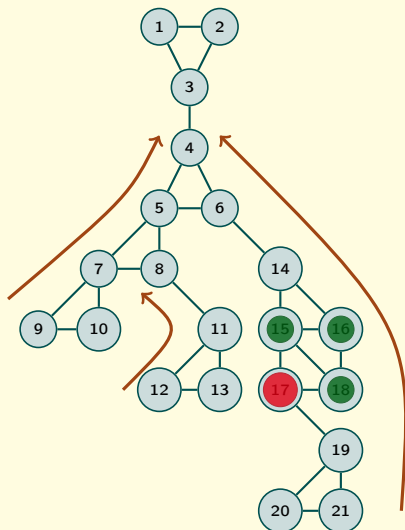


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Tree-width = minimal number cops  
monotonously capturing Robber – 1

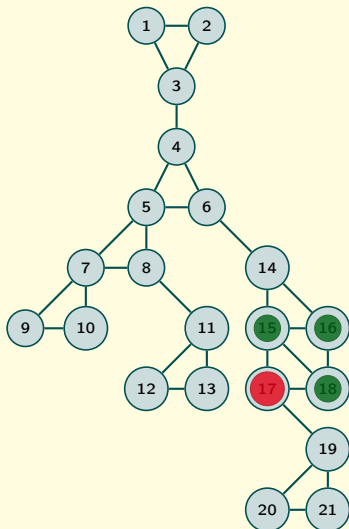
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Path-width:

the same,

but robber invisible.

## DAG-width

- ▶ straightforward generalisation to directed graphs  
(= tree-width on undirected)
- ▶ the robber can run only **along directed paths**

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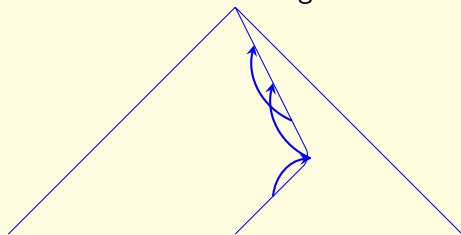
Compare to other generalisations:

- ▶ Kelly-width: the robber is **invisible** and **inert**
- ▶ directed tree-width: the robber **locked in his component**
- ▶ bounded DAG-width: only parity games

## Bounded and Unbounded DAG-width

Bounded: only few paths in one direction

- ▶ DAGs (1)
- ▶ trees with short back-edges



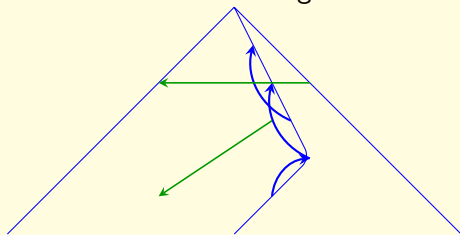
Unbounded:

- ▶ Grids
- ▶ trees with many long back-edges

## Bounded and Unbounded DAG-width

Bounded: only few paths in one direction

- ▶ DAGs (1)
- ▶ trees with short back-edges and horizontal connections



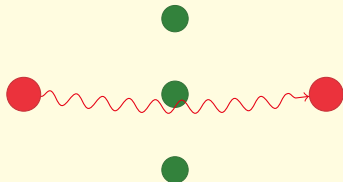
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## Generalisation to Multiple Robbers

Same rules, but multiple robbers:

- ▶ all  $r$  robbers must be captured
- ▶ robbers can **jump** to each other
- ▶ global monotonicity: no robber can access a vertex that was inaccessible to **all** robbers



# Main Result

## Theorem

*one robber captured by  $k$  cops*

$\Rightarrow$

*$r$  robbers captured by  $k \cdot r$  cops*

Rough proof idea:

- ▶ a team of  $k$  cops against each robber
  - ▶ play independently with each team
1. This works for tree-width. . .
  2. This needs improvement for directed case. . .

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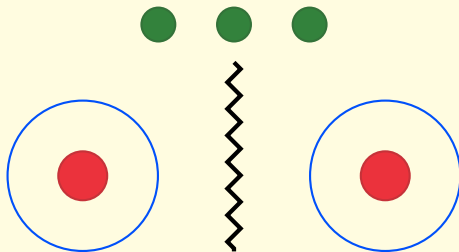
## Theorem

For tree-width:

*one* robber captured by *k* cops

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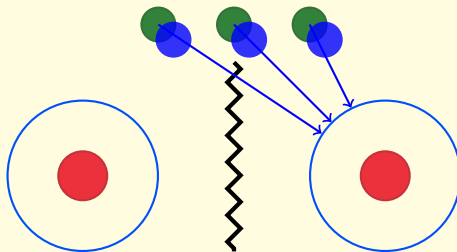
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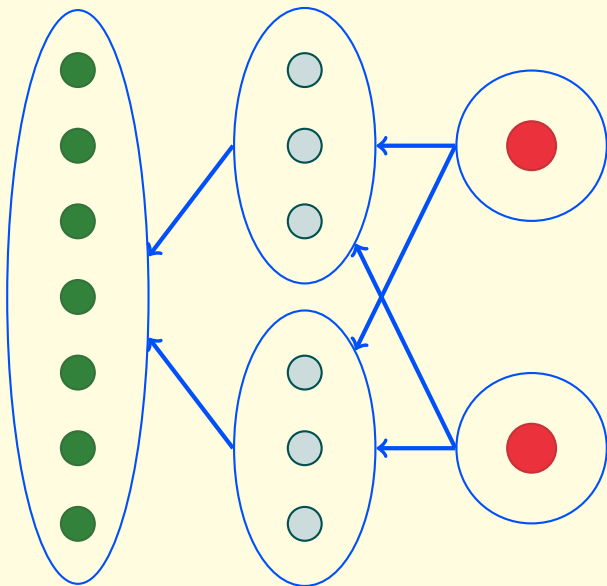
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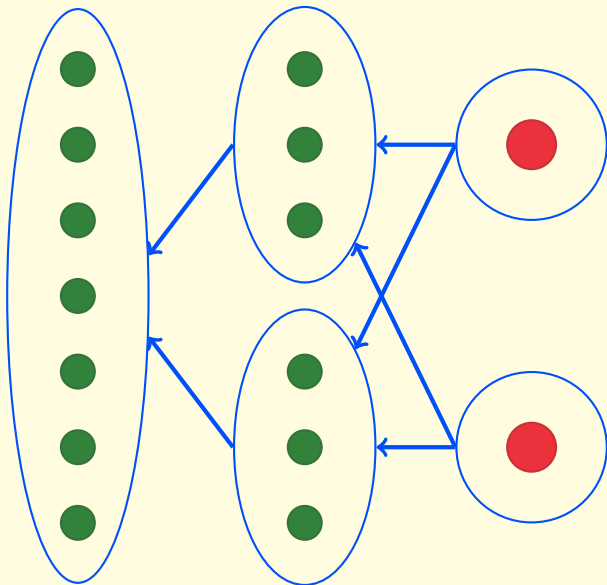
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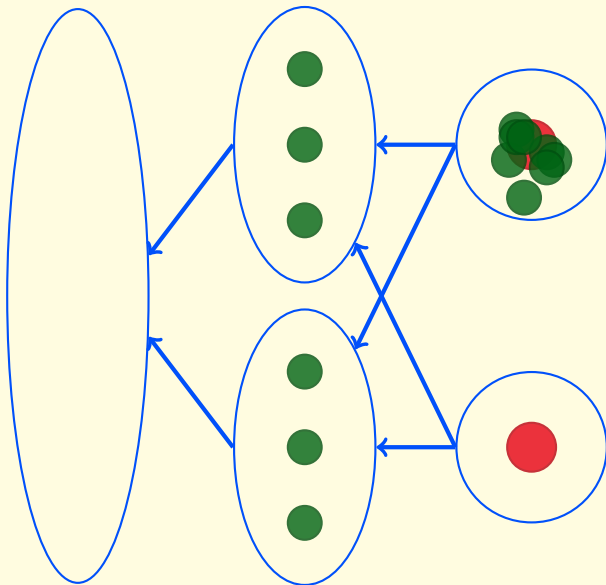
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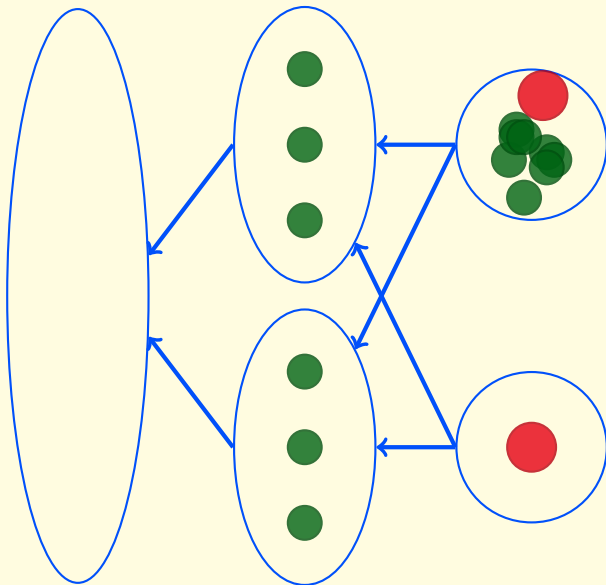
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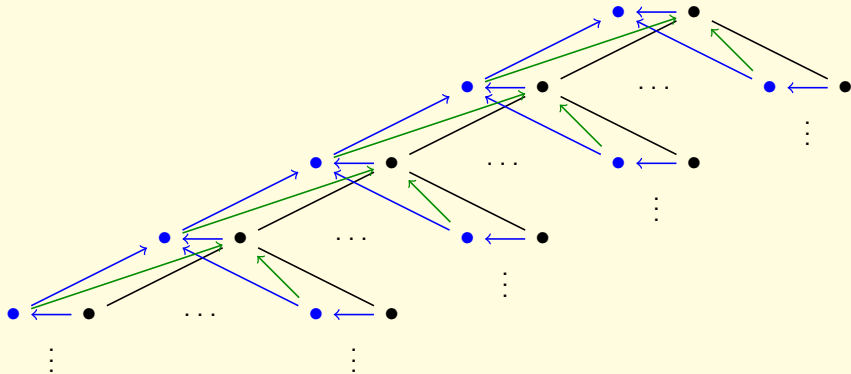


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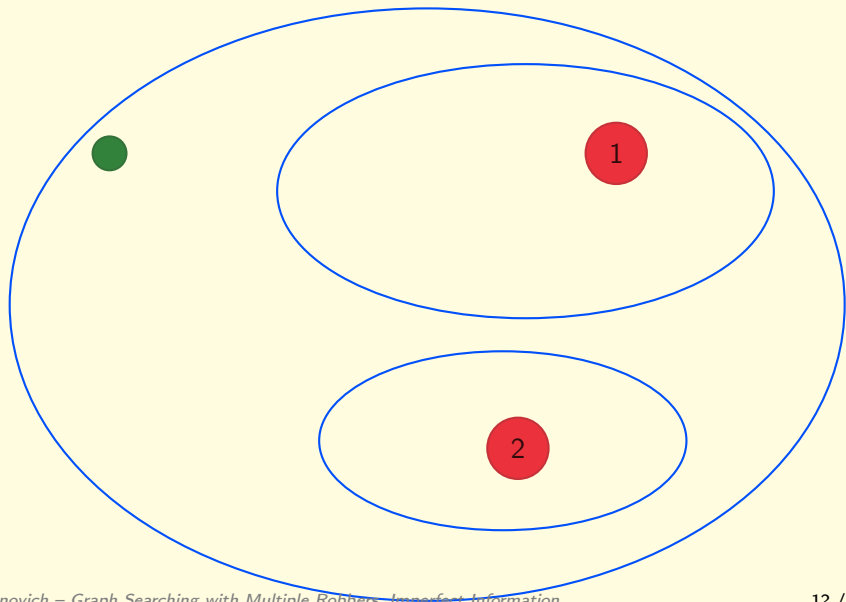


# Offhanded Cops

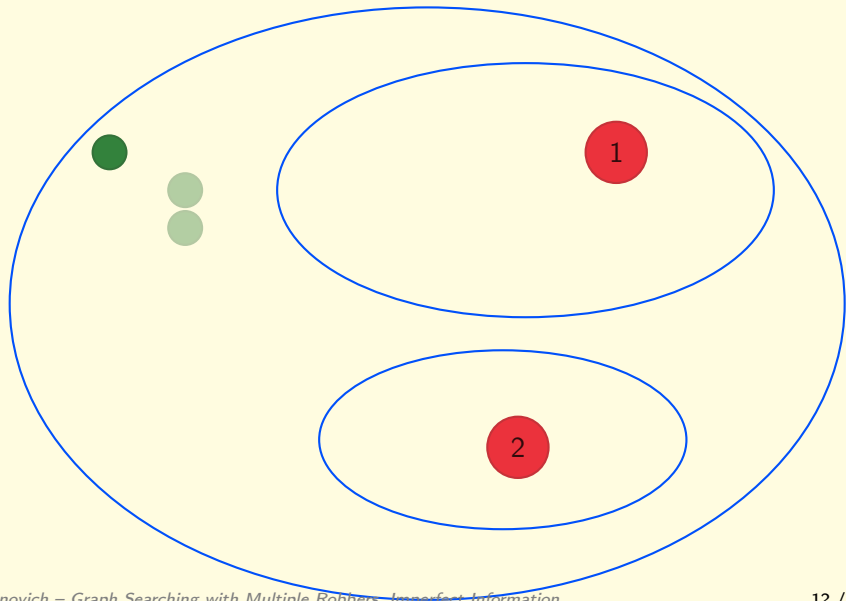
Never place a cop outside robber's component.



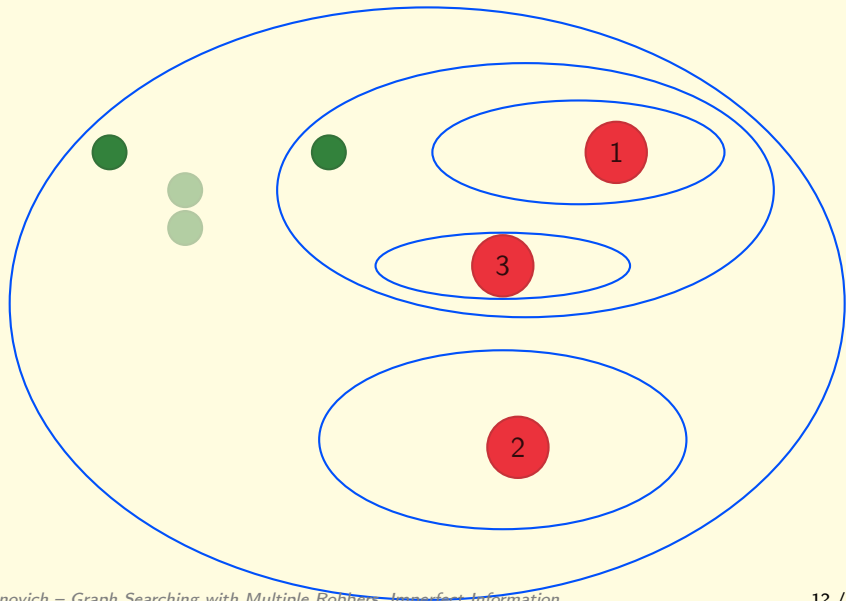
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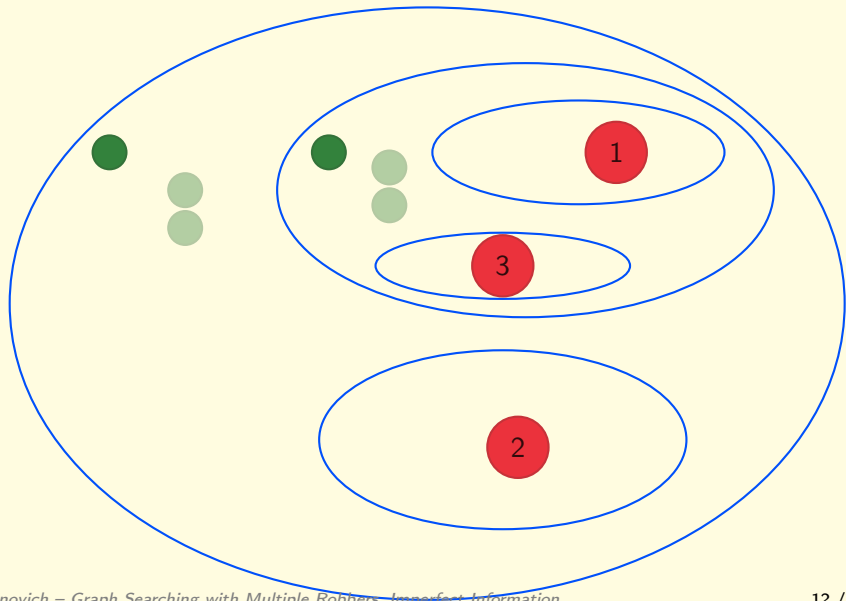
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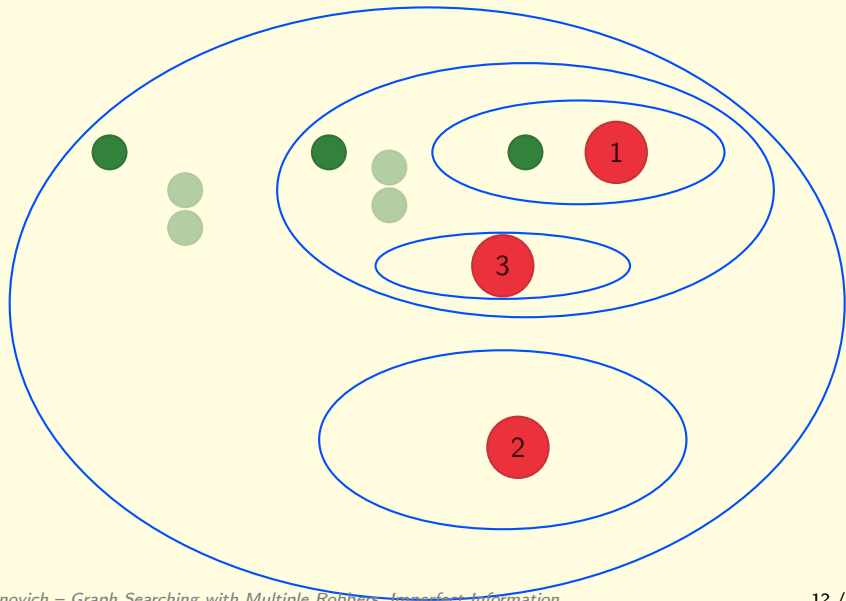
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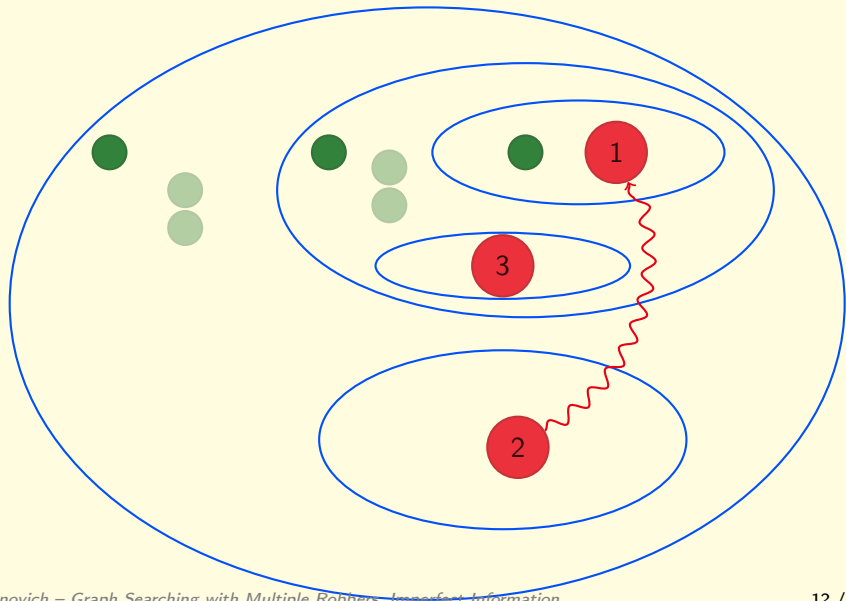
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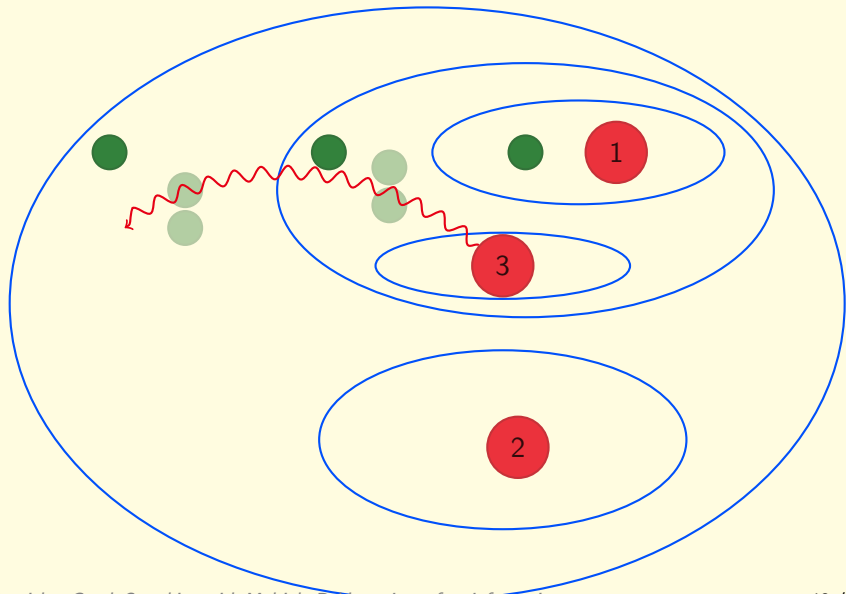
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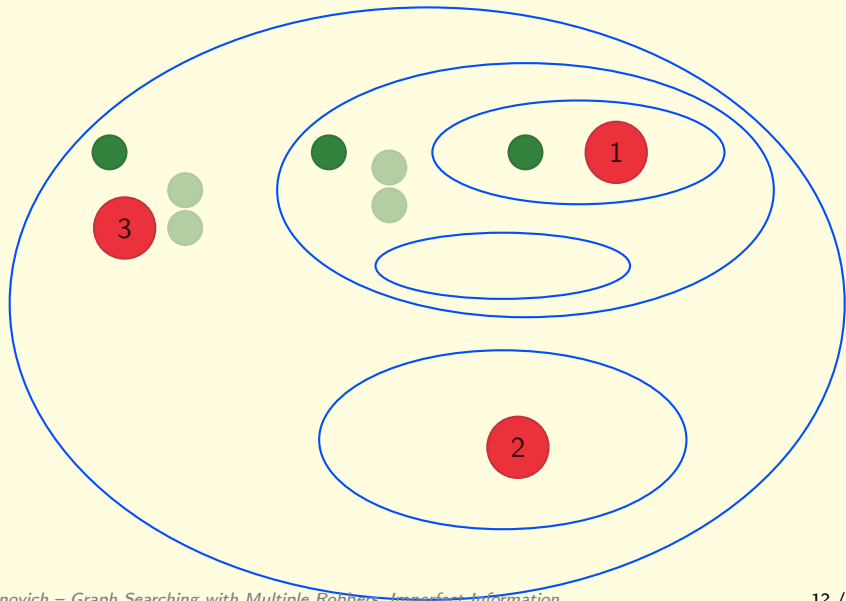
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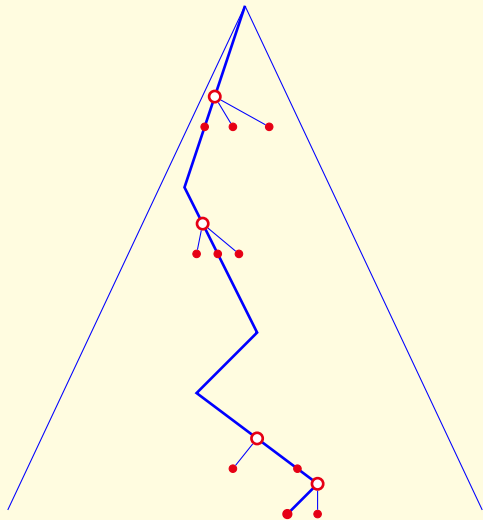
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## Maintaining and linearizing several plays



# Corollaries

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## Theorem (Robbers Hierarchy)

*There are classes of graphs on which every new robber demands new cops until directed path-width is reached.*